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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

New Hybrid Interpolation Method for Motion Transfer in Fluid–Structure Interactions

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Introduction

IN computational aeroelastic analyses by loosely coupled approaches, efficient and accurate interpolation is essential to transfer structural deformation to aerodynamic grid points due to the intrinsically different resolution in fluid and structural grids.

The numerous interpolation methods used in motion transfer can be categorized into two groups: surface spline and tracking methods. The surface spline methods solve a linear equation consisting of radial basis functions to obtain a globally representative surface. For example, these methods employ infinite plate spline[1] (IPS), thin plate spline[2] (TPS), and multiquadrics[3] (MQ). IPS is widely used because of its simplicity and smoothness. The system matrices from these methods are prone to becoming ill conditioned when used with unstructured grids and large numbers of nodes. A subdomaining approach has been applied to TPS and MQ to reduce this instability.[4] The tracking methods use the natural coordinates of fluid nodes projected to nearest structural elements. The accuracy of these methods is low at highly curved geometries. Several algorithms such as quadratic interpolation, initial distance vectors, and rotated distance vectors with point normals have been suggested to increase accuracy.[5]

In this study, a new hybrid interpolation method, referred to as curvature gradient index local fitting (CILF), is suggested. CILF uses a surface spline method at local geometries having high curvature gradient and a tracking method at the remaining geometries. Analytical test cases and motion transfer of an AGARD 445.6 wing[6] are implemented to show efficiency, accuracy, and smoothness of the suggested method in comparison with other interpolation methods.

New Hybrid Interpolation Method

A new hybrid method referred to as CILF uses a surface spline method for local grid regions where the curvature gradient index is satisfied by a criterion and a tracking method for the rest of the grid. TPS without a subdomaining approach is used for the surface spline method of CILF. TPS was indicated as a robust, cost-effective, and accurate method in Ref 7. A tracking method with initial distance vectors[5] is employed for the tracking scheme of CILF.

Curvature Gradient Index and Criterion

In CILF, the curvature gradient indices of a structural element can be calculated from the variation of dihedral angles at a time step

$$CI_e^l = \cos^{-1}\left(\boldsymbol{n}_e \cdot \boldsymbol{n}^l\right)\Big|_k^{k+1} \tag{1}$$

where $l = 1, 2, 3$ (for a triangular element) and where CI_e^l denotes a curvature gradient index corresponding an adjacent element of a structural element (e), k is a time step index, and $\boldsymbol{n}_e$ and $\boldsymbol{n}^l$ are unit normal vectors of the element and its adjacent element, respectively.

The absolute value of the curvature gradient can be used as a criterion. Alternatively, a relative criterion can be determined as a percentage of the structural elements used in three-dimensional local TPS. For this, all of the curvature gradient indices are sorted as an array in descending order.

Three-Dimensional Local TPS

A three-dimensional local TPS uses element-based local splines with a small data set, comprising of all adjacent nodes of the two end nodes on the criterion-satisfied edges of a structural element. A local TPS can be achieved by the following steps:

1) Project nodes (data set) to a plane parallel to a structural element.
2) Calculate distances d from the nodes to the projected nodes p.
3) Form the three-dimensional A matrix and $\boldsymbol{b}$ vector in a linear equation

$$A\boldsymbol{a} = \boldsymbol{b} \tag{2}$$

where $\boldsymbol{a}$ is a spline coefficient vector,

$$A = \begin{bmatrix} K & P \\ P^T & 0 \end{bmatrix}, \qquad \boldsymbol{b} = [d_1 \quad \cdots \quad d_m \quad 0 \quad 0 \quad 0 \quad 0]^T$$

$$K = \begin{bmatrix} K_1\left(p_{1_x}, p_{1_y}, p_{1_z}\right) & \cdots & K_m\left(p_{1_x}, p_{1_y}, p_{1_z}\right) \\ \vdots & & \vdots \\ K_1\left(p_{m_x}, p_{m_y}, p_{m_z}\right) & \cdots & K_m\left(p_{m_x}, p_{m_y}, p_{m_z}\right) \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & p_{1_x} & p_{1_y} & p_{1_z} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & p_{m_x} & p_{m_y} & p_{m_z} \end{bmatrix}$$

$$K_i(x, y, z) = (1/16\pi)\phi(x, y, z)\,\ln\phi(x, y, z)$$

$$\phi(x, y, z) = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$$

and m is the number of the nodes projected on the plane.

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4) Obtain interpolated distances of all fluid nodes on the structural element by

$$f(x, y, z) = \sum_{i=1}^{m} a_i K_i(x, y, z) + a_{m+1} + a_{m+2}x + a_{m+3}y + a_{m+4}z \tag{3}$$

Tracking with Initial Distance Vectors

In the tracking method with initial distance vectors, the displacement of fluid nodes can be tracked by the structural nodal displacements from the natural coordinates using a neighbor-to-neighbor search algorithm.[8] In the process of the search algorithm, the fluid nodes are projected to nearest structural elements with initial distances due to the unmatched structural and fluid grids.

Results

Analytical Test Cases

To demonstrate the accuracy and efficiency of the suggested method, four two-dimensional test functions are chosen as follows:

$$W_1(x, y) = (1 + 9x^2 + 16y^2)^{-1} \tag{4}$$

$$W_2(x, y) = 1 - [(x^2 + y^2)/2]^{\frac{1}{2}} \tag{5}$$

$$W_3(x, y) = 1.5xe^{-x^2-y^2} \tag{6}$$

$$W_4(x, y) = [1.25 + \cos(5.4y)]/[6 + 6(3x - 1)^2] \tag{7}$$

where $-1.5 \leq x, y \leq 1.5$. In this work, the fluid grid comprises 1968 nodes and the structural grid has 128 nodes irregularly distributed on a plate. The criterion for CILF is set to an absolute value, ±5 deg. To evaluate the accuracy of CILF compared to IPS and tracking method with initial distance vectors, the root-mean-square errors are calculated. In terms of accuracy, the rms errors of CILF are much smaller than those of tracking as shown in Table 1. In terms of efficiency, CILF is much faster than the IPS method as shown in Table 2.

Table 1 Comparison of rms errors of test functions

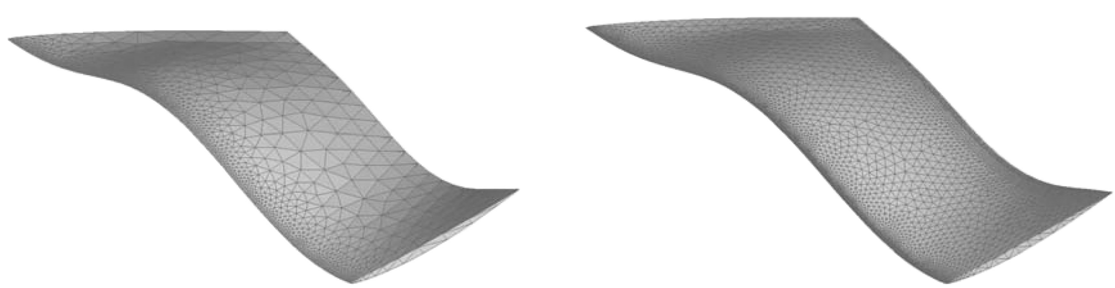
Test functions	Tracking	IPS	CILF
$W_1(x, y)$	0.0124	0.0041	0.0048
$W_2(x, y)$	0.0065	0.0026	0.0036
$W_3(x, y)$	0.0112	0.0007	0.0023
$W_4(x, y)$	0.0110	0.0033	0.0039

Table 2 Comparison of relative CPU time of test functions

Test functions	Tracking	IPS	CILF
$W_1(x, y)$	0.16	3.80	1
$W_2(x, y)$	0.10	2.30	1
$W_3(x, y)$	0.07	1.74	1
$W_4(x, y)$	0.14	2.12	1

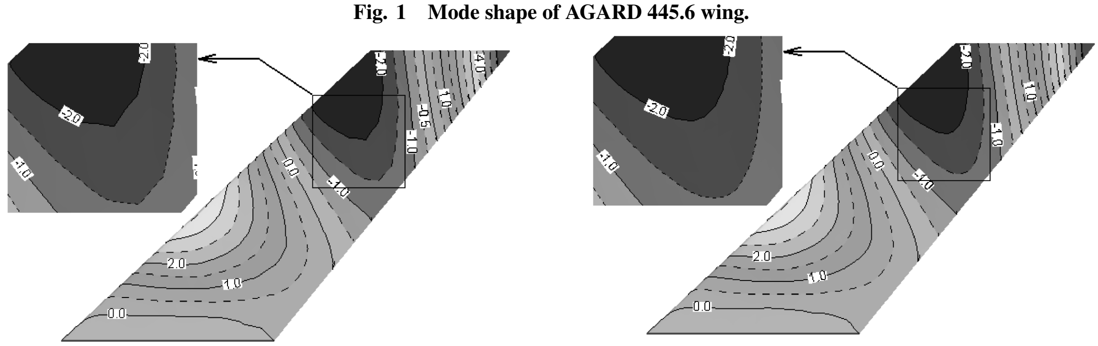
Table 3 Comparison of CPU time

Methods	Relative CPU time
Tracking	0.32
IPS	130.48
CILF	1.00

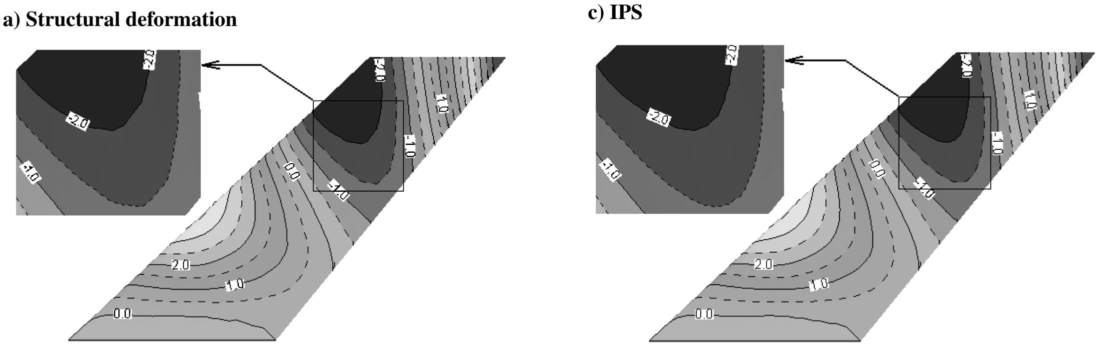


a) Structural grid **b) Interpolated fluid grid by CILF**

Fig. 1 Mode shape of AGARD 445.6 wing.

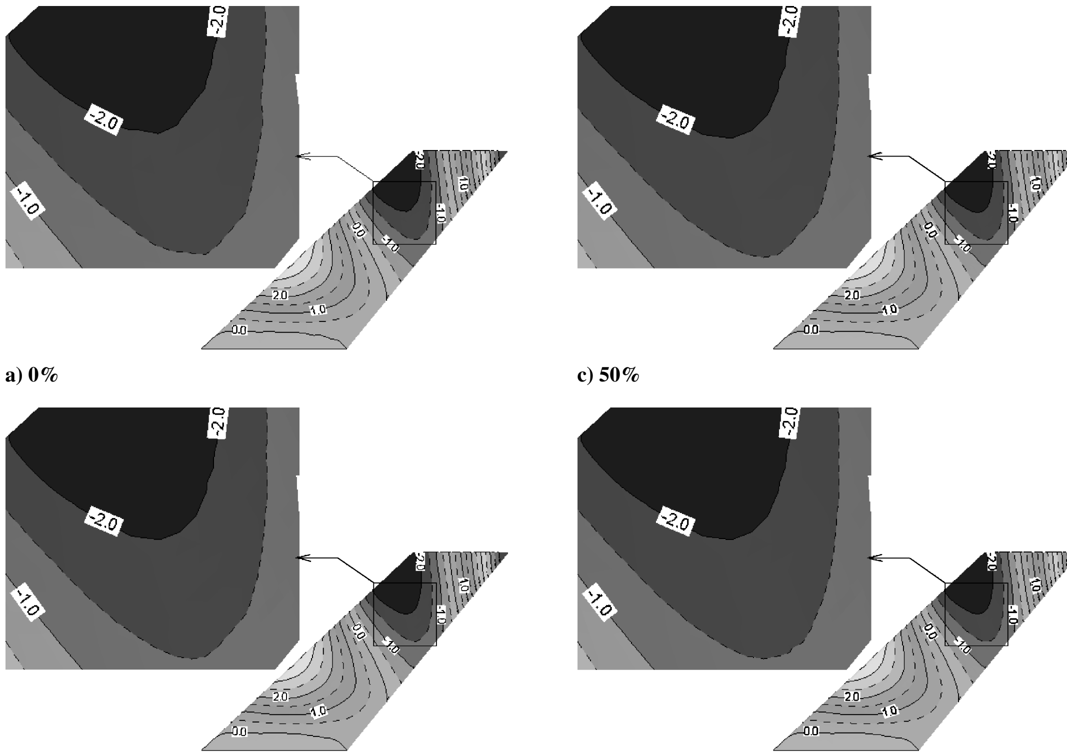


a) Structural deformation **c) IPS**



b) Tracking **d) CILF**

Fig. 2 AGARD 445.6 wing deformation contour.



a) 0% **c) 50%**

b) 25% **d) 75%**

Fig. 3 Smoothness by relative criteria.

Table 4 CPU time by relative criteria

Structural elements, %	Relative CPU time
0	1.000
25	1.865
50	2.506
75	3.130

AGARD Wing Deformation

The motion transfer of the fourth vibration mode of the AGARD 445.6 wing is performed. The unstructured fluid and structural grids enclose the actual three-dimensional wing surface with 35,870 and 3282 nodes, respectively. Figure 1 shows interpolated fluid grids based on the structural deformation using CILF. Figure 2 shows that CILF and IPS provide smoother contour lines than the tracking method. The CPU time in Table 3 shows that CILF is 130 times faster than IPS. The effect of the relative criterion on smoothness is shown in Fig. 3. The CPU time required varies approximately in proportion to the value of the criterion, as shown in Table 4. The relative criterion can, thus, be chosen by considering the tradeoff between smoothness and efficiency for a given problem.

Conclusions

The motion transfer of complex three-dimensional aeroelastic wings can be performed using the proposed interpolation method. The results presented show that the suggested hybrid method of combining the efficiency of tracking and the smoothness of splines provides efficient, accurate, and smooth interpolations in comparison to the traditional methods discussed in this paper.

Acknowledgments

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